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ON OBSERVING MATHEMATICS TEACHER LEARNING

ALF COLES

The focus of this article is on mathematics teacher learning. In a characteristic enactivist move, my interest is not in ‘what is’ mathematics teacher learning (there will be no typology or classification of knowledge) but rather, ‘what happens when I say that I observe learning or knowing’? Two major research fields within mathematics education are currently the study of language and the study of teacher knowledge. It is a peculiarity that there has been little work bringing these fields together. The article has a methodological focus; I argue, with some illustrative data, that the perspective of enactivism is suited to the study of talk in a manner that allows an analysis of mathematics teacher learning (where ‘talk’ here might include transcript data from a meeting, or an interview or questionnaire). I begin with an invitation to notice your own observations of learning, in the transcript below.

Observing a new awareness

The following conversation took place during a session in a part-time Master’s course for teachers (more detail of the context will be given later on). One teacher (Susie [1]) expresses what I take to be a new awareness. Susie begins this transcript by describing her use of a framework for encouraging student talk (which was a focus for an action research project she was undertaking). The framework of talk moves she used was presented (in Firmender *et al.*, 2014) in a linear manner (repeat and check; think time; add on; agree/disagree and why; partner talk). The transcript begins with Susie explaining to the group the first four parts of the framework and how she interprets them. Susie had given the participants in the session a transcript from a recording she made in a lesson in which she was using the framework of talk moves.

In all the transcripts, standard punctuation is used to hint at flow and intonation.

- 1 *Susie*: Yes. ‘You’re [*her students*] going to tell me something; I’m going to repeat it. Then I’m going to allow you some thinking time together; then I’m going to see what the other person thinks. I’m going to ask you whether you agree or disagree.’ Like that.
- 2 *Alf*: I don’t see that framework as a linear sequence. Is that how you’re interpreting it?
- 3 *Susie*: Yes! [*said loudly, followed by laughter*] You don’t see it as a linear sequence. Wacky! [*laughs again*] Massive! Thanks, Alf. No wonder. It doesn’t have to be a linear sequence.
- 4 *Clare*: But even in here [*pointing to Susie’s transcript from a lesson*] you’ve done a lot of step one, before you managed to go to step two.
- 5 *Susie*: I know. I was still thinking of it as a linear sequence because it talks about talk moves; move from there to there to there. It’s not, no.
- 6 *Alf*: Just that the move is used for students and it could be any strategy.
- 7 *Susie*: Okay. Thanks, Alf. [*laughs*] I’m alright now. I’m sorted.
- 8 *Alf*: That’s why talking is so important. [*laughs*]
- 9 *Rick*: You can still use the script, it’s just been shifted around a bit.
- 10 *Susie*: I can, yes.
- 11 *Alf*: And for what’s appropriate.
- 12 *Susie*: Do you know what’s hilarious? My thing was: check, add-on, engage multiple, agree or disagree. I even wrote it linearly in here. I could only see it that way. Move on now. I’m happy!

As I will set out below, from an enactivist perspective, there is evidence here of learning and some of the energy and excitement of Susie’s talk can perhaps be guessed at from the transcript. In fact, in a later session Susie reflects that, while she is now much more effective in getting her students talking (by

using the framework of talk moves flexibly) something is lost by not going through a sequence, in that she is not forced to do unfamiliar steps. No learning or awareness is ever fixed and in making some choices we blind ourselves to others. Susie described having got to a more complex position where she can make a choice about either applying the framework in a linear manner or not. I see the transcript above as strong evidence of mathematics teacher learning in contrast to what, perhaps more usually, can be taken to indicate learning. After some theoretical considerations, I will return to further transcript data to tease out other kinds of evidence of learning.

What do I observe when I say I observe learning?

From an enactivist stance ‘All doing is knowing and all knowing is doing’ (Maturana and Varela 1987, p.27). Knowing can therefore be acknowledged through observing action; learning can be defined as a change in knowing and therefore a change in acting. For example, faced with the same question if I answer inadequately at Time 1 and adequately at Time 2, then this can be taken as evidence of learning. One of enactivism’s roots is in phenomenology and the study of first person awareness. I follow Gattegno (1987) in taking awareness to be that which enables action (see, Mason 2018). Any learning, or change in ‘doing’ in a context, implies a ‘re-seeing’ (however minimal) of that context and this re-seeing is a new awareness (which may or may not be consciously recognised). So, I can observe new actions in a context and potentially observe the ‘re-seeing’ that is a new awareness.

On my own journey to becoming a mathematics teacher, I recognise a complex layering of awarenesses that slowly led to more effective action. There was a need for many behaviours to become automatic, for instance: knowing students’ names; the use of technology; the use of school ‘behaviour management’ routines; ways of working on the mathematics before a lesson, in order to be able to hear what students say. Beginning teaching, a child coming up to me to tell me something about their homework at the start of a lesson might have triggered (in

me) stress and worry about starting the lesson calmly and on time. After some years, I experienced the same kind of incident as indicating there was perhaps a useful conversation to be had with the whole class about the homework. It is not possible, now, to tease out the different awarenesses that enabled the different response over time (the mark of learning); the enactivist story of this difference would be one that entails change in the 'me' that is being created by the world around me, and change in the world around me that I am creating. Perception is a *bringing forth of a world* and learning implies a co-ordination (or co-evolution) of self and world, as each responds and adapts to the other. The circularity of this world-view has implications for the doing of research.

In a commentary article to a recent ZDM issue on enactivism as a methodology, Simmt and Kieren (2015) identify one of the key 'moves' common to research in mathematics education that is done from an enactivist perspective, as 'the observer' (p. 308). This move starts from Maturana's insight (1988) that, 'everything said is said by an observer to another observer' (p. 9). In every aspect of being, what is said is said by an observer, but that does not imply that, in every aspect of being, the role of the observer is acknowledged or taken into account. The shift in perspective entailed by this move is captured by Maturana when reflecting on what prompted his insight:

In 1965 when I was studying color vision in pigeons I realized that I could no longer pretend that one saw the colors as features of an external world, and that I had to abandon the question, "how do I see that color?" and ask instead, "what happens in me when I say that I see such a color?" (Maturana, 2002, p. 5)

For research from an enactivist perspective, a similar shift in focus is required, towards questions that are about ourselves and our awareness, in relation to the focus of study. It is a commonplace that any questionnaire or interview will give access to what participants *say* about certain phenomena, not to the phenomena themselves. Any classroom video can give access to what was said or done, not to what participants may or may not have been experiencing. Perhaps harder, though, is to notice times when we make a shift, for example, from our

interpretation of *what* we hear said in a video, to our interpretation of *why* we think it was said. When I say I observe learning, I am paying attention to difference over time and my awareness of differences in actions or awareness.

What might mathematics teacher learning, or knowing, look like in talk?

In an overview article on language and communication in the field of mathematics education, Morgan *et al.* (2014) identify four categories of research:

- analysis of the development of students' mathematical knowledge
- understanding the shaping of mathematical activity
- understanding processes of teaching and learning in relation to other social interactions
- multilingual contexts' (p. 846)

This article proposes an additional strand, which is: 'analysis of the development of mathematics teacher knowing', for instance in the context of professional development, and I hope to show that enactivism is well suited to such an analysis.

An enactivist perspective on language links the activity of languaging more closely to action than to meaning (in contrast to a more constructivist perspective, *e.g.*, Anghileri, 1995). Or, to put this another way, meaning is not an internal event (nor is it purely 'external') but rather, meaning arises through interaction with the world and is an expression of co-ordinated action, or co-action. Words and languaging are mechanisms for co-ordinating meaning. On this view, therefore, language arises and functions as a co-ordination of co-ordination of action.

Languaging always and already places us in relation to each other. In the context of considering multi-lingual classrooms, Geiger, Margolinas and Straßer (2017) point to what they describe as the 'indigenous perspective of taking responsibility for bringing people together' (p. 18). An awareness of languaging as a relational act can perhaps prompt such a sense, of 'bringing people together' through talk.

I am interested in observing the learning of mathematics teachers through their talk and what happens when I say that I observe learning. Noticing changes in talk implies a recognition of when talk might be said to have stayed ‘the same’. A central feature of the enactivist world-view is an assumption about the existence and importance of patterns and hence the significance, for analysis, of identifying pattern. The insight of ‘the observer’ move of enactivism is important here; patterns are not assumed to have causal power. Any pattern noticed is noticed by an observer. As an enactivist researcher, I am interested in patterns that others can also observe (and so standard practices of triangulation, for example, can be important), but there is always the recognition that a pattern arises through interaction and the original noticing of a pattern says as much about the person doing the noticing as the thing observed. In other words, there is no suggestion that other people looking at the same data will observe the same patterns as I do; however, once noticed, it should be possible for others to see those patterns as well, for instance the recurrence over time, or across speakers, of a particular word or form of words.

In Coles (2015), I set out five tentative mechanisms to guide an enactivist study of language. The five elements are: (1) recursive inquiry; (2) equifinality; (3) the systematic search for pattern; (4) micro-analysis; (5) meta-communication. These were set out in some detail in the earlier paper and it is not my intention to reprise the detail of arguments there. Taken together, these mechanisms can provide a way of analysing learning in talk as the making of new distinctions. The notion of a ‘distinction’ is taken from the work of Spencer Brown (1972) and links to enactivist insights about observation. Any observation takes place within a ‘marked space’ that inevitably brings with it an ‘unmarked space’ that we are not aware of in the moment of observation. The boundary between these spaces is the ‘distinction’ (that is therefore inherent in any observation), this distinction also being unavailable to awareness in the moment of observation. We can observe the observations of others (and ourselves over time) and notice distinctions and

marked spaces (these observations themselves also carrying unmarked spaces and un-noticed distinctions). For instance, if a teacher tells me they cannot find a way of explaining to their students why $-1 \times -1 = 1$, I might hypothesise a distinction (*i.e.*, something un-noticed, carrying with it an unmarked space) that, for this teacher, teaching mathematics is only about explaining. We can only guess at distinctions and unmarked spaces of others (and ourselves); but making explicit possible interpretations can provoke energetic responses (*e.g.*, in the form of commitment to try out new behaviours in the classroom).

By using transcript data to analyse learning (as is common in studies of language) we may only have the verbal channel available for study but I do not want to commit myself to the position that the only form of learning is via language. The purpose of this article is to identify some general characteristics of the study of mathematics teacher talk and, in doing so, perhaps to point to some un-noticed distinctions.

With these thoughts in mind, in the next section, I offer an analysis of mathematics teacher talk, in order to draw out differences around the kinds of talk that would provide evidence of learning, or, more precisely, evidence for observation of learning. Some of this learning is in relation to teaching mathematics and some, given the context, in relation to researching mathematics classrooms.

Some categories in analysing learning

Having laid some theoretical groundwork for an enactivist analysis of mathematics teacher talk, this section draws on empirical data from meetings of a group of mathematics teachers in the context of a Master's in Mathematics Education module. The teachers had a common interest in 'widening participation' of their students in the study of mathematics and in participation in higher education more generally. The project [2] ran over two years, with two cohorts of teachers (eight in one group and ten in the second). The participants were all mathematics teachers from either primary or secondary phases, teaching in state schools in the Bristol area in the UK. They

were volunteers on the project and the project paid for their fees, for the module they were taking. That module supported the teachers in undertaking an action research study in their own classrooms. The Master's sessions (ten for each cohort) took place at University and were opportunities for these teachers to report to the wider group on what they had been doing. Each teacher chose a focus for their action research that was relevant to their classroom. These included: developing reasoning in the classroom; co-operative learning; increasing student participation; Singapore bar modelling; working on 'higher' level content (*e.g.*, trigonometry) with adolescents who have a history of underachievement in mathematics. In Year 1, two University tutors (of whom I was one) and a researcher were present along with the teachers; in Year 2, there was one tutor (myself) and one researcher.

The role of the University tutors in the group could be seen in enactivist terms, in that we were giving priority to the articulation of observations. The explicit aim, during meetings, was occasioning participating teachers to engage in a recursive process of action in their classrooms and reflections on those actions. Meetings followed a pattern, allowing teachers time for raising issues and for joint reflection on each other's work. My own analysis of patterns of communication adds a further layer of observation to the work that took place. I am reporting here on new awarenesses I made through this process.

All meetings were audio-recorded. This project was chosen for analysis because there was suitable audio data for an analysis of teacher talk, *i.e.*, there was nothing atypical about the groups, compared with previous ones on this Master's module. The audio-recordings of all meetings were transcribed. I have worked on the transcripts with a range of different foci and so knew some of the data well. For the purposes of this article, I read through all the transcripts again but this time looking out for moments where either there was evidence of a teacher responding differently to the same situation, or evidence of a new awareness being made or having been made. This process was consistent with the idea of a systematic search for pattern

(Coles, 2015). With such a focus in mind, all instances were highlighted and then considered in isolation from the rest of the text.

There were four patterns that emerged: a report of a change in behaviour; a report of a new awareness; an observed change in response over time; and, an observed new awareness being made in the moment. The possibilities are summarised in Table 1.

	Change in behaviour	New awareness
Teacher self-report		
Observation in talk		[see transcript from the start of the article]

Table 1: Possibilities for noticing learning through talk

I will take each box in turn and offer illustrative transcriptions. Of course, the categories are not as neat as the table suggests and, as will be seen below, there are overlaps and blurrings. I am conscious the word ‘mathematics’ does not appear in the list above; this absence is perhaps explained by the fact that the main focus of meetings was on researching mathematics teaching (e.g., refining a research question, collecting data, analysing data) in order to improve some aspect of students’ learning of mathematics. It is also important to note that this table in no way attempts to classify all teacher talk, rather, only those moments where it might be possible to say ‘I observed learning’. Neither am I wanting to suggest communication can be neatly packaged into categories; rather, with a particular focus on a particular definition of learning (given above) these are some patterns I can notice about my observations. In discussing the observations, below, I shift to the pronoun ‘we’ as, significantly, I am claiming that the patterns I point to are observable by others.

Self-report of a change in behaviour

The example below is a verbal report, from a teacher, of a change. Teacher Paula (a teacher of grade 5 students) had a focus on developing students’ verbal reasoning skills.

Paula: I find it fascinating [pause] the more it [my action research study] has progressed, the more it's about this community of learning, we're in it together, I hardly get hands up any more [pause] but very rarely do I get 'I'm stuck', they turn to the person they've had a debate with [pause] it's like they realise that they're on the same par, thinking ground, and ask 'what did you get?', across the room. I feel like I've come a long way. It used to be asking them to stop shouting across the room [pause] but 'you can talk about maths across the room' [pause] now it naturally comes and they self-monitor [pause] it only happens at specific times, and in dribs and drabs while they're working. The environment in my classroom is more community based. (Cohort 1, Meeting 8)

Paula's statement "I feel like I've come a long way" is a report of a change (in this case linked to a new approach to teaching and learning mathematics). The change that she is describing here took place before the meeting and, as listeners or readers, we have little access to what Paula means by having come "a long way", nor what a classroom being "community based" might mean. This kind of evidence of change or learning is, from an enactivist perspective, equivalent to what might be gained from a questionnaire where, without further probing, there is little access to the criterion by which change and learning is being judged to have taken place. The change appears powerful and meaningful for Paula, but the only access we get to what she says she now does differently is that she allows children to "talk about maths across the room". One important distinction to point to here is that, as researchers, from the transcript it is not possible to observe the change or learning being discussed. We have no access to criteria against which to judge the reported change, nor any associated new awareness; rather, what we observe is a report about change by the speaker.

Self-report of a new awareness

A slightly different form of evidence of learning, still reported, is when a teacher offers some criteria for the reported change,

i.e., we are given some access to what is different and what new awareness occasioned the change. One teacher, Francis, was working with a single student (aged 10) on learning number using a particular set of manipulative shapes and video recording sessions with the student. Francis had stated in meeting 1, in relation to the use of manipulatives, that from his experience, ‘it does work’.

Francis: I’ve found that there are certain activities where he gets really confused with the shapes, and others where he finds it easier. *[Some dialogue skipped]*

Francis: Regarding the use of manipulatives, do they support this child in learning his addition facts? That’s kind of my idea. I’ve been going through the different activities, making notes, and I’m wondering if this *[manipulative]* helps him to understand or remember.

Alf: You said there are times where it’s helping, and times where it’s not. How are you making that distinction?

Francis: I was thinking that if the shapes help him understand, or even recall some number facts, then they do support him.

Alf: How would you tell that the shapes are helping him do that?

Francis: In some of the activities it’s very clear that they don’t, and he gets confused. Later in this video there is a moment where he suddenly understands because he can see it, and by manipulating the shapes, he can understand it. That’s when I realised there are certain activities where the shapes do help this child.

At the start of this transcript, Francis articulates a new awareness, in now questioning when manipulatives do or do not help. I push for some more detail. Francis responds by pointing to moments when the manipulatives support the recall of a number fact (*e.g.*, as opposed to having to count to confirm that fact). I push again for how Francis might tell it is the manipulatives that are making the difference, and Francis responds that it is very clear from the video. Soon after, in the meeting, Francis shows a video extract where the student

answers an addition problem (what must you add to 2 to make 8), using the shapes and without counting.

The talk here can be seen as a report on Francis's learning, in relation to his developing a more nuanced view of the use of manipulatives. Although still a report on learning, unlike in the section above, we have some access to the change in Francis's thinking in the form of a new awareness (sometimes manipulatives do not help). Furthermore, what it means for a manipulative to "help" is related to whether it supports a learner answer a question without needing to count, something potentially observable by others. In this instance, implicit within the report of a new awareness are likely to be new behaviours as a teacher, indicating some blurring or even nesting of categories on the top row of Table 1.

Observing a change in behaviour

At the start of the first group meetings, a question I used was: 'what is action research?'. All participants had come having read a chapter of the set text (Altrichter *et al.*, 2007) and were going to be undertaking action research studies in their own classrooms. The first transcript, below, is taken from the first meeting (of Cohort 2); the second transcript is from the next meeting of the same group, a fortnight later. Together these transcripts offer evidence of change for Hannah.

Alf: What is action research? [*Some dialogue skipped*]

Mila: I thought of it as a circular motion [*pause*] keep reflecting, keep improving [*pause*] [*Some dialogue skipped*]

Hannah: We can slightly change, to make the students react [*pause*] to how the classroom environment is situated

In the second meeting, I return to the same question as at the start of the transcript above, since not everyone in the group had been present at Meeting 1.

Alf: Where are you at in terms of what action research is, what do we mean by this funny word, this process?

Mila: When I've been thinking about it, I think it's kind of a circular motion, think about something, study it, see how you can improve it, trial it, repeat the process, circular trying how it goes. If it all goes to pot, tweak it a bit, see what happens, hopefully find a magic mix that'll work. *[Some dialogue skipped]*

Hannah: It's a communicative way of going through the ideas, whether it be teaching assistant to teacher or student to teacher, it's that triangular mode of everybody getting the right feedback so everybody's happy with the process and the ways that we're learning.

What can be observed in these two transcripts is a shift in the response of Hannah compared with, say, Mila. For Mila, what makes action research is consistent across Meetings 1 and 2, in being linked to a circular process of trying something out, reviewing it, 'tweaking' your ideas and repeating. In contrast, in Meeting 1, Hannah links action research to making some change (as a teacher) in order to 'make the students react'; but in Meeting 2, Hannah expresses the idea that action research involves communication and feedback among students, teaching assistants and teachers, in a 'triangular mode', to ensure everyone is happy with the ways they are learning.

In the change in Hannah's response, we can observe stronger evidence of learning than in the earlier two sections which were reports of learning. This is not a judgment about the qualities or depths of learning across the different cases, but only to notice that the evidence for that learning was less strong than here, where we can observe a difference in response to the same prompt. From the transcript, we do not know if Hannah was aware, herself, of any shift in her thinking, whereas the examples of self-reports necessitate the teacher being aware of a change they have undergone.

Observing a new awareness

The final example of evidence of learning was the one reported at the start of the article. It comes from Cohort 1 and is the clearest example from this category across the two sets of meetings. Referring back to that transcript, in turn 12, the 'move

on' refers to the structure of the meetings where each participant would be given 'time' to talk through where they are at in their action research study. Susie indicates that she does not need any more time, presumably since she has had a new awareness she wants to work on. The shift in Susie is from seeing her framework as a sequence of strategies she must use in a linear order to promote classroom talk, to viewing the framework as set of strategies to be used flexibly as appropriate in the classroom. Observation of a new awareness necessarily unfolds in front of the observer, in this case in relation to Susie's use of the framework: 'it doesn't have to be linear'. In turns 3, 5 and 12, Susie articulates the shift in her awareness. It could be argued that we also observe, in the transcript, a change in Susie's behaviour in relation to the framework she uses. Observing the arising of a new awareness perhaps implies there will be new behaviours and hence a nesting of the bottom row of Table 1, as with the top row.

Implications

This article set out to provide some methodological categories that would allow an analysis of observations of mathematics teacher learning through talk. These categories may be relevant to a critical analysis of what is claimed as evidence of learning, within mathematics education, for example, studies in which teachers' self-reports of beliefs are taken to *be* their beliefs. In contrast to approaches to studying language that deliberately avoid analysis of learning, enactivism offers a perspective that allows something to be said about knowing and changes in knowing (which are interpreted as learning). In particular, I claim there are four ways we might observe teacher learning, via talk:

- 1) a teacher reporting a change;
- 2) a teacher reporting on a new awareness;
- 3) observing a change in response to the same prompt;
- 4) observing a change in awareness.

From an enactivist perspective, these categories, though potentially overlapping, are in a hierarchy in terms of the

strength of evidence for observation of learning (but, to reiterate, not in terms of the significance of that learning itself). The nesting of categories suggests a less definite arrangement than the list might suggest (Table 2).



		 <i>stronger evidence</i>	
		Change in behaviour	New awareness
 <i>stronger evidence</i>	Teacher self-report	<div>Listeners are offered a description of a change in behaviour, in general terms.</div>	<div>Listeners are offered a description of a change and an associated new awareness.</div>
	Observation in talk	<div>Listeners notice a new response to the same stimulus, compared to an earlier instance.</div>	<div>Listeners observe a new awareness being articulated, in that moment, and what is being distinguished.</div>

Table 2: Different kinds of evidence from observations of learning

As well as providing a tool for analysis, these categories potentially also help the awareness of the facilitator of professional learning of mathematics teachers. I might want to design opportunities to offer the same stimulus in order to observe any changes in response. As a facilitator, I might attune myself to whether a teacher is, say, reporting on a change without giving us access to what they did or what new awareness they had. I can begin to make categorisations *about* the talk of others. My own learning is a learning about the learning of others. It feels important to notice moments of new awarenesses being made, both to ensure there is space in discussion for these to play out and, potentially, to comment to the rest of the group on what we have just observed, as a powerful moment of learning. My hope is that this article may have pointed to some distinctions and un-marked spaces around what we mean when we say we have observed mathematics teacher learning.

Notes

[1] All teachers' names are pseudonyms.

- [2] Funding for the project reported here was provided by the University of Bristol's Widening Participation Research Fund (project title: 'Removing the mathematical barriers to widening participation in Higher Education').

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